

Analysis of Student Conceptual Errors and Procedural Errors in Solving Mathematical Problems

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Article Info

Article history:

Accepted: 23 September 2023

Publish: 04 Oktober 2023

Keywords:

Conceptual Errors

Procedural Errors

Mathematics

Abstract

This study aims to analyze conceptual and procedural errors in solving mathematical problems. This research is a descriptive qualitative research that aims to explore mathematical misconceptions in students and their views on the misconceptions they experience. The misconceptions in this study were obtained from the results of tests and interviews. Misconceptions are most common in geometric materials, but they are not significantly different from other materials. Students still have difficulty due to lack of conceptual understanding of the problem or difficulty connecting mathematical concepts. The type of misconception that is most often experienced by students is confusion because they do not master the supporting concepts. The misconception that is most rarely experienced by students is the wrong classification of concept examples.

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1. INTRODUCTION

Mathematics has a substantial contribution in the development of human life, especially in problem solving reasoning (Mtetwa et al., 2011). In solving mathematical problems, students must understand the concepts related to these problems (Woolfolk & Margetts, 2013; Zahid & Sujadi, 2017). Concept learning is very important to be given in learning in schools at every level because it underlies learning and cognitive development (Senemoğlu, 2013).

Mathematics is an abstract science. Students will best understand abstract theory when they have a solid understanding of what is being abstracted (Gillian, 1990). In understanding these abstract concepts, students construct mathematical concepts in the classroom and relate them to their life experiences (Campbell & Campbell, 2008; Welder, 2012). Students must be able to reorganize their experiences in a personal mathematical form so as to make students more proficient and understand abstract complex forms (Ndemo & Mtetwa, 2015).

Mathematical misconceptions occur at every level of education ranging from elementary, secondary to higher education (Ojose, 2015). Students still experience confusion in some mathematical concepts (Roselizawati et al., 2014). This misconception occurs in various branches of mathematics such as algebra, arithmetic, geometry, calculus and probability. In algebra, misconceptions occur because students' understanding is limited (Ndemo & Mtetwa, 2015). In geometry, students experience misconceptions in distinguishing parallelograms, trapezoids, rhombuses, and similar shapes (Al-Khateeb, 2016). Students experience misconceptions in fractional operations (arithmetic) and exponential addition (algebra) (Ojose, 2015). Misconceptions also occur at the university level. Students still have difficulty due to lack of understanding of the concept of the

problem or difficulty connecting mathematical concepts mathematics (Kharis et al., 2018). Students also still experience errors in the concept of probability theory (Astuty, 2015). Students also do not know how to apply rotational transformation. Students understand the algebraic calculations but do not understand the meaning of the transformation itself (Winarso & Toheri, 2017). Misconceptions occur not only in students but also in teachers. The misconceptions that occur in the teacher are: (1) Misconceptions on the concept of numbers, (2) Misconceptions in the order of operations, (3) Misconceptions on fractions, (4) Misconceptions on indexes, (5) Errors due to weakness mastery of technical vocabulary, and (6) Misconceptions in algebra (Naseer, 2015).

The fact that there are still misconceptions among students and teachers is a concern in learning mathematics. It will be very dangerous if students move from one level to the next with the wrong misconception (Ojose, 2015). When students experience misconceptions from the start, there will be bigger misconceptions at the next level (Roselizawati et al., 2014). Errors and misconceptions if they are embedded in students' memory will be difficult to remove (Sarwadi & Shahrill, 2014; Svandova, 2014). It is difficult to eliminate misconceptions, but it is mandatory for teachers to know why students make mistakes so that these mistakes do not happen again (Ndemo & Mtetwa, 2015).

Many studies reveal students' misconceptions and errors in mathematics (Biber et al., 2013; C. S & SM, 2018; Mohyuddin & Khalil, 2016; Ozerem, 2012; Rakes, 2010; T & ECM, 2019). This research will examine students' misconceptions in terms of various branches of mathematics, namely algebra, arithmetic, geometry, and probability. The formulation of the problems raised in this study are as follows: (1) How do students view misconceptions in mathematics?, (2) What are students' misconceptions in mathematics?. The aims of this study are as follows: (1) To explore students' views on misconceptions in mathematics, (2) to explore what students' misconceptions are in mathematics.

2. METHOD

This research is a descriptive qualitative research that aims to explore mathematical misconceptions in students and their views on the misconceptions they experience. The misconceptions in this study were obtained from the results of tests and interviews. The test was conducted to reveal how the misconceptions that occur in students. The test questions consist of four core mathematics materials, namely geometry, probability theory, algebra and calculus. Each material consists of three description questions. The total test questions given are twelve questions. In addition to tests, interviews were also conducted to obtain data on students' views of the misconceptions they experienced.

This study reviews the misconceptions based on the misconception indicators from Rochmad et al. (2018). The misconceptions consist of: (1) the concept of an incorrect definition; (2) incorrect use of the concept; (3) wrong classification of concept examples; (4) misinterpretation of the concept with the meaning of the concept, (5) confusion due to not mastering the supporting concepts, and (6) connecting the wrong concepts. The following is a recapitulation of the occurrence of misconceptions in answering the test questions given.

3. RESULTS AND DISCUSSION

This study aims to analyze conceptual and procedural errors in solving mathematical problems. The test was conducted to obtain data on conceptual and procedural errors in solving mathematical problems. Interviews were conducted to obtain more in-depth information about conceptual and procedural errors made by students.

The test consists of general mathematical material, namely geometry, probability theory, algebra and calculus. The use of some of these materials aims to obtain representative data regarding conceptual and procedural errors in students. Each material consists of three test questions. The total test questions given to students are 12 test questions.

Table 1. Recapitulation of Misconceptions in Answering Test Questions (TQ)

Students	Subject											
	Geometry			Probability Theory			Algebra			Calculus		
	QQ 1	QQ 2	QQ 3	QQ 1	QQ 2	QQ 3	QQ 1	QQ 2	QQ 3	QQ 1	QQ 2	QQ 3
S1	√	√	-	√	√	-	√	√	√	√	√	√
S2	√	√	-	√	√	√	-	√	√	√	√	√
S3	√	√	√	√	√	√	√	√	√	√	√	√
S4	-	√	√	√	-	√	√	√	√	√	-	-
S5	-	√	√	√	-	√	√	√	√	√	-	√
Total	3	5	3	5	3	4	4	5	5	5	3	4
	16			12			14			12		

Overall, the overall procedural understanding of the subject is better than conceptual understanding. This can be seen from the work on the test questions where in terms of conceptual understanding, the subject tends to make a lot of mistakes. This is inversely proportional to the question of procedural understanding where the subject made a few mistakes.

Conceptual errors in this study are viewed from indicators of conceptual understanding, while procedural errors are viewed from indicators of procedural understanding. The following is a recapitulation of conceptual and procedural understanding based on indicators from Wawan et al. (2017).

Table 2. Indicators of Conceptual Understanding

Items	Indicators	Students					Total
		S1	S2	S3	S4	S5	
1.	Restate a concept	-	√	-	-	√	2
2.	Classify mathematical objects based on their properties	-	√	√	√	-	3
3.	Give an example of a concept	-	-	-	-	√	1
4.	Presenting concepts in various forms of mathematical representation	√	√	-	√	-	3

In the table above, it can be seen that in conceptual understanding, the indicators most controlled by the subject are indicator 2 and indicator 4. Indicator 2 states that the subject can classify mathematical objects based on their properties. Indicator 4 states that the subject can present concepts in various forms of mathematical representation. The indicator that is most rarely mastered by the subject is indicator 3. Indicator 3 states that the subject can be able to provide an example of a concept. Only one subject meets this indicator. The other three subjects made an error on this indicator.

In indicator 1, there are three subjects who made mistakes. The subject made a mistake in re-expressing a concept. This can be seen in the following image.

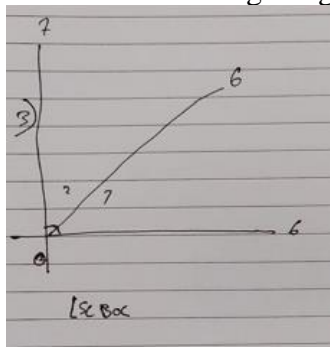


Figure 1. Conceptual Understanding Errors in Rephrasing a Concept

The picture is the result of the S3 test. It is seen that the S3 is not able to re-express the concept of angled angles. S3 doesn't understand that the total size of the angles in a right angle is $.90^0$

In indicator 2, there are two subjects who made mistakes. Subjects make mistakes in classifying mathematical objects based on their properties. This can be seen in the following image.

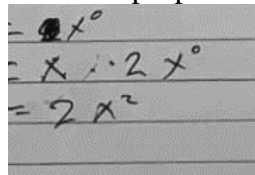


Figure 2. Errors in Conceptual Understanding in Classifying Mathematical Objects based on their Properties

The picture is the result of the S5 test. It can be seen that S5 is not able to classify degrees and exponents. Degrees represent the measure of an angle while exponents represent exponents. This causes both objects to be inoperable. This is the mistake that the S5 made in operating on two different objects.

In indicator 3, there are four subjects who made mistakes. The subject made an error in providing an example of a concept. This can be seen in the following image.

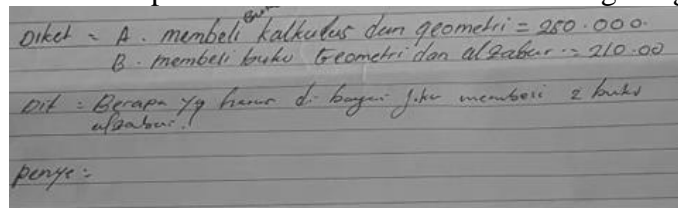


Figure 3. Errors in Conceptual Understanding in Giving an Example of a Concept

The picture is the result of the S4 test. It can be seen that S4 is not able to determine that the problem in this problem includes an example of a two-variable linear equation system.

In indicator 4, there are two subjects who made mistakes. Subjects make mistakes in presenting concepts into various forms of mathematical representation. This can be seen in the following image.

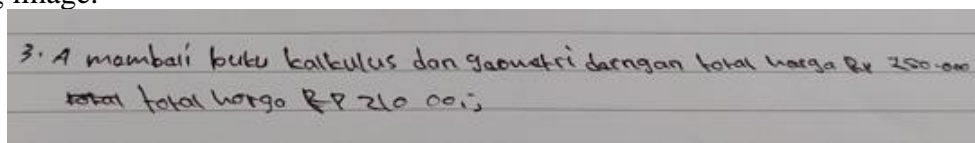


Figure 4. Errors in Conceptual Understanding in Presenting Concepts into Various Forms of Mathematical Representations

The picture is the result of the S3 test. It can be seen that S3 is not able to present the concept in the form of mathematical sentences, namely in algebraic form.

Table 3. Indicators of Procedural Understanding

Items	Indicators	Students					Total
		S1	S2	S3	S4	S5	
1.	Write down the steps in solving math problems (knowledge of algorithms)	√	-	√	√	√	4
2.	Explain the reasons for each step of solving the problem that has been determined.	-	√	√	√	√	4

Procedural understanding of the subject is classified as better than conceptual understanding. This can be seen from only one subject who made an error in indicator 1 and one subject who made an error in indicator 2. In indicator 1, S2 made a mistake in writing down the steps in solving math problems. This relates to knowledge of algorithms. The error made by S2 is shown in the following figure.

$$y + z = \text{Rp. } 20.000$$

$$y = \text{Rp. } 40.000$$

$$\text{Rp. } 40.000 \times 2 = \text{Rp. } 80000$$

Figures5. Procedural Comprehension Errors in Writing Steps in Solving Math Problems

If we look at the step-by-step problem solving procedure, there are errors at each step. Each step is not related to each other. As a result, a single solution that satisfies these conditions will never be found. This is a very fatal procedural misunderstanding.

In indicator 2, S1 made a mistake in explaining the reasons for each of the predetermined problem-solving steps. The error made by S1 is shown in the following figure.

$$\angle BPP = \frac{1}{2} \cdot a \cdot t$$

$$= \frac{1}{2} \cdot 6 \cdot p$$

$$= \frac{6}{2} p$$

$$= 3p$$

Figure 6. Errors in Procedural Understanding Explaining the Reasons for Each Steps to Solve Problems That Have Been Determined

The picture shows S1's error in solving the given problem. The problem contains an order to measure the size of an angle. S1 has understood that what to look for is the size of the angle, but S1 uses the wrong concept. S1 uses the formula for the area of a triangle to find the measure of an angle. However, the angles and the area of a triangle are different concepts. S1 stated that he did not understand the concept. In the interview session, S1 stated that according to him the problem would be solved quickly by using the triangle formula.

Conceptual errors as shown in the results of the analysis of student answers above because students do not understand what concepts are appropriate to solve the problems posed. Interviews with several students showed that students were not sure they had used the right concepts in problem solving. There are also cases where students are unable to distinguish between the concepts of probability and combinations, resulting in errors in applying the right concepts to problem solving. this finding is in accordance with the findings of Kharis et al. (2018). that students still have difficulty due to lack of understanding of the concept of the problem or difficulty connecting mathematical concepts. Students also still experience errors in the concept of probability theory (Astuty, 2015).

The results of the interview show that the errors that occur are due to the lack of understanding of students in the concepts of the material, so that the lack of understanding results in students applying the wrong concepts in solving mathematical problems.

4. CONCLUSION

Overall, misconceptions are most common in geometric materials, but they are not significantly different from other materials. Students still have difficulty due to lack of conceptual understanding of the problem or difficulty connecting mathematical concepts. The type of

misconception that is most often experienced by students is confusion because they do not master the supporting concepts. The misconception that is most rarely experienced by students is the wrong classification of concept examples.

5. RECOMMENDATIONS

Further researchers are expected to be able to examine the factors behind the occurrence of misconceptions. This is very important so that we can find out what can cause misconceptions and find solutions.

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