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Cognitive Analysis of the Understanding of Euclidean Geometry Concepts of Students in the Mathematics Education Study Program

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Abstract

This study aims to conduct a cognitive analysis of mathematics education students' understanding of concepts in Euclidean geometry. Students are often unable to fully achieve and optimize the formal cognitive stages required in Euclidean geometry, resulting in a gap between the complex structure of the material and their thinking abilities. This is a descriptive qualitative study aimed at examining students' cognitive processes. The subjects were 11 second-semester Mathematics Education students who had taken the Euclidean Geometry course in the even semester of the 2024/2025 academic year. The main instrument was a Euclidean geometry comprehension test supported by a questionnaire to explore thinking processes and misconceptions. Evaluation of understanding was reviewed based on the cognitive level. The results showed that many students, even at the tertiary level, were still at a low level in the Van Hiele framework, namely level 0 (Visualization) or level 1 (Analysis). This condition causes students to tend to experience difficulties in conducting formal proofs; 63.6% of students lacked understanding due to a low logical understanding of Euclidean postulates. The level of students' understanding of Euclidean geometry is strongly influenced by how they process information cognitively. Therefore, teachers need to develop structured learning strategies based on Van Hiele's learning phases to facilitate students' cognitive progress in a gradual and planned manner.

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1. INTRODUCTION

The main foundation in learning mathematics that is deductive and systematic begins with the study of Euclidean geometry. Euclidean geometry plays a crucial role in developing the logical and analytical thinking skills students need to face the challenges of subsequent geometry courses. Mathematics education students need to be equipped with an understanding of the structure and properties of Euclidean geometry, which is not only dependent on conceptual mastery but is also strongly influenced by the student's cognitive developmental stage. Therefore, studying the relationship between understanding Euclidean geometry and cognitive level is an important focus, especially for prospective educators and mathematics education students.

International studies show that understanding geometric principles is part of humankind's natural cognitive intuition. Humans possess a basic understanding of the concepts of straight lines and angles without formal education, supporting the assumption that cognitive aspects play a significant role in understanding geometry from an early age (Izard et al., 2011).

National studies have seen a significant increase in research examining the relationship between students' thinking styles and their ability to comprehend geometry. A study (Fajriah & Amalia, 2017) showed that cognitive styles significantly influence students' geometry problem-solving strategies. Furthermore, research (Hasratuddin et al., 2020) demonstrated that the application of cognitive conflict in geometry learning can encourage students to shift from visual understanding to formal deductive thinking.

Cognitive development theorists such as Jean Piaget state that individuals at the formal operational stage are capable of abstract and deductive thinking, which is an essential foundation for understanding the logical structure of Euclid's geometry (Piaget, 1972). However, mathematics education students are still often unable to fully achieve and optimize this cognitive stage (Prasetya Rini et al., 2021; Raharjo et al., 2025; Ramadhan et al., 2022). This creates a gap between the complex structure of geometric material and students' thinking abilities. Difficulties in using concepts in their use, stating the meaning of terms representing the application of concepts, are often found even at the elementary school level (Fauzi & Arisetyawan, 2020).

Van Hiele's theory provides a more specific perspective on understanding the development of geometric thinking. Van Hiele and Hiele-Geldof (1958) proposed five levels of geometric thinking: visualization, analysis, informal deduction, formal deduction, and rigor. Each level depends on learning experience, not just age, gender, and educational level (Raharjo et al., 2024; Rusmana, 2021; Zakiah, 2020). In other words, students who have not yet reached the formal deductive level will encounter many obstacles, one of which is difficulty understanding the axiomatic structure of learning Euclidean geometry.

With a primary focus on learning evaluation, Bloom's Taxonomy provides a cognitive framework that allows lecturers to assess not only learning outcomes but also the thinking processes that play a crucial role in student evaluation. Bloom classifies cognitive processes into six levels: remembering, understanding, applying, analyzing, evaluating, and creating (Wilson, 2016). The use of this taxonomy can be a more precise measurement to explore the depth of students' understanding of Euclid material.

Based on the above explanation, it is clear that understanding Euclidean geometry cannot be separated from the cognitive development of students. Unfortunately, most previous research is still limited to quantitative measurement of learning outcomes through final tests, without exploring the cognitive processes that occur during the learning process. Research that focuses on the cognitive level in understanding Euclidean geometry is still rarely conducted in depth, especially in higher education environments. Therefore, this study aims to conduct a cognitive analysis of students' understanding of concepts in Euclidean geometry. This study is important as an effort to fill the research gap related to various approaches that have been studied in the context of geometry in general. Thus, it can fill the existing gap by examining how the cognitive level of Mathematics Education students plays a role in understanding the concepts and structures in Euclidean geometry.

2. METHOD

This research is a descriptive qualitative study that aims to examine students' cognitive processes in understanding Euclidean geometry. Descriptive qualitative research is a type of research that aims to understand phenomena in depth through descriptions based on qualitative data (Creswell, 2014). Descriptive qualitative research focuses on how researchers describe and interpret processes, events, or interactions that occur in a particular context (Bogdan & Biklen, 2007). The research subjects were second-semester students of the Mathematics Education study program who had studied Euclidean geometry.

The subjects of this research are 11 students in the Mathematics Education Study Program, semester I, who have taken the Euclidean Geometry course. The subject is selected randomly by purposive sampling based on the results of an initial diagnostic test that measures basic understanding of Euclidean geometry and potential misconceptions (Patton, 2015; Saunders-Russell, 2016). All subjects had a background of geometry learning experience since high school, but with different intensities and approaches in the learning process. Subject selection was based on varying levels of initial understanding to obtain rich data on cognitive processes.

The main instrument of this research is the **Euclidean geometry comprehension test,** designed to cover aspects of Euclid's concepts, definitions, and theorems. This test is intended to stimulate students' reasoning at various Van Hiele levels. In addition, it is also used as a **questionnaire** to explore students' thinking processes, problem-solving strategies, and misconceptions (Cohen et al., 2017).

The data collection process is carried out through three main stages:

1. **Written Test:** Initial data collection used comprehension tests to identify students' initial cognitive levels.
2. **Observation Questionnaire:** Used to trace students' level of thinking during the work process, as well as collect data about their beliefs and attitudes towards geometric proof (Whittle et al., 2014; Darma, 2019).
3. **Semi-Structured Interview:** Interviews were conducted with selected subjects after the test, aiming to delve deeper into the reasons behind their answers (method*think-aloud*indirectly) and validate the cognitive processes that occur (Seidman, 2013).

The tests relate to understanding of Euclid's concepts, definitions, and theorems. Observation questionnaires are used to track students' thinking levels. They also include observations of student learning processes and discussions.

Evaluation of students' understanding of Euclidean geometry is reviewed based on **cognitive level** to measure their understanding comprehensively. Qualitative data analysis follows an interactive model that includes the following stages: **data condensation**, **data presentation** (via summary tables and graphs), and **conclusions drawing** (Miles, M. B., Huberman, A. M., & Saldaña, 2014)

Data analysis focused on interpreting the findings based on two theoretical frameworks:

1. **Revised Bloom's Taxonomy:** To classify students' cognitive processes (remembering, understanding, applying, analyzing, evaluating) in Euclid's Geometry. (Wilson, 2016)
2. **Van Hiele Model:** To analyze students' geometric thinking levels (Visualization, Analysis, Informal Deduction, Formal Deduction), which is the main framework for concluding their cognitive development in this subject.

3. RESULTS AND DISCUSSION

Empirical research results indicate a significant gap between the demands of the Euclidean Geometry curriculum and students' actual cognitive abilities. This phenomenon is reflected in the questionnaire data grouped by cognitive level as follows.

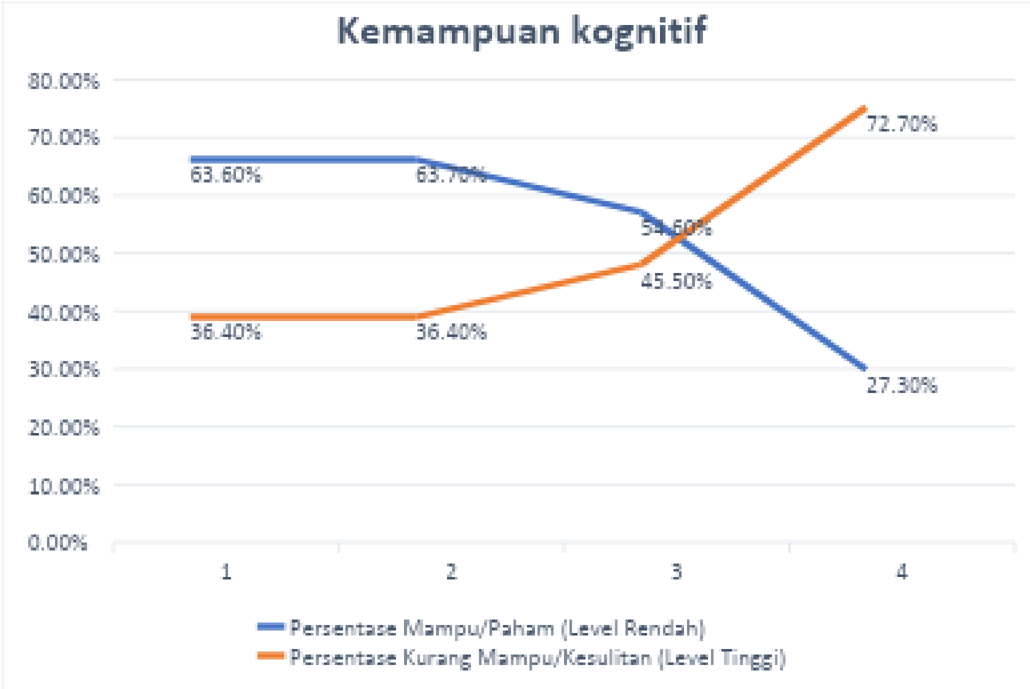

Figure 1. Cognitive Ability Graph

This data is reinforced by a specific finding, where 63.6% of students showed a lack of adequate logical understanding when asked to explain Euclid's five postulates formally (as a prerequisite for proof).

This discussion analyzes the empirical findings by integrating them into a theoretical framework. **Van Hiele's theory** is used to explain the causes and implications of students' cognitive difficulties.

3.1 Consistency at the Low Cognitive Level

The data shows that students demonstrate adequate abilities (around 63% ‘able/understand’) in aspects that only require thinking in **Level 1 (Analysis).** At this level, they are able to identify and use the properties of geometric shapes separately, as seen in their success in applying the concepts of angles and triangles (Nurhasanah et al., 2017). However, this ability is not sufficient for Euclidean Geometry. In line with the opinion **of Hiele** (1958), the learning process must be designed to help students reach higher levels, because mastery at one level is a prerequisite for understanding the next level.

3.2. Major Obstacles to the Transition to Formal Deduction

The peak of the research findings lies in the difficulties students have in **Constructing Logical Proof**, where **72,7%** are in the 'Less Able/Difficulty' category. This confirms that the majority of students **trapped** at the Analysis level and have not yet succeeded in achieving **Level 3 (Formal Deduction)**.

This difficulty is supported by findings (Fajriah & Amalia, 2017), which show that cognitive style has a significant influence on geometry problem-solving strategies. Students who still rely on visualization (Level 0) will have difficulty constructing formal deductive arguments because their reasoning is still tied to specific case examples, not to universal logical structures. Failure to reach Level 3 means students cannot understand the role of axioms and theorems as a logical basis, so that proofs are only considered as memorization, not as a reasoning process (Hasratuddin et al., 2020). Based on findings (Shodikin et al., 2023), students' analogical reasoning in the visualizer cognitive style can improve cognitive performance compared to the verbalizer cognitive style. This can be considered a measure of student cognitive ability.

3.3. Didactic Implications

These findings imply that previous teaching methods likely did not explicitly facilitate transitions between cognitive levels. Based on Van Hiele's theory, teaching should follow five phases of instruction (Inquiry, Direct Orientation, Explication, Free Orientation, Integration) to encourage cognitive leaps. Lecturers need to implement strategies that can trigger these transitions of **cognitive conflict**, forcing students to question their visual assumptions and move to abstract logical reasoning (Piaget, 1972).

Van Hiele's cognitive level model provides a powerful framework for analyzing student understanding in Euclidean geometry courses. This theory states that individuals, including mathematics education students, must progress through a series of hierarchically structured levels of thinking to achieve deeper geometric understanding. Research using this model often shows that many students, even at the tertiary level, remain at lower levels, namely level 0 (Visualization) or level 1 (Analysis), where their understanding relies heavily on physical visualization or the identification of isolated properties (Maulid et al., 2024; Qur'ani et al., 2024; Setiyawan et al., 2024; Umami & Asdarina, 2024). This phenomenon is an important topic of discussion because it indicates a gap between curriculum demands, which often require formal deductive reasoning, and students' actual cognitive abilities. Therefore, the use of Van Hiele's model is crucial in diagnosing learning problems and designing appropriate pedagogical interventions aimed at helping students move toward higher levels of thinking.

The low level of understanding of Euclidean geometry among students, as discussed in the context of Van Hiele's theory, has significant implications for learning and teaching. Students who are only at the visualization level tend to struggle with formal proofs because they lack a logical understanding of the relationships between properties, which is characteristic of the formal deduction level (Juwantara Ridho, 2019; Zakiah, 2020). This lack of ability can hinder them from mastering the basic concepts of Euclidean geometry and complicate the transition to non-Euclidean geometry, which requires more abstract thinking. Discussion of these findings suggests that cognitive development does not automatically occur with age but rather depends heavily on the quality of instruction received. Therefore, lecturers need to design a structured learning strategy based on Van Hiele's learning phases (Inquiry, Direct Orientation, Explication, Free Orientation, Integration) to encourage students' cognitive development. The focus should be directed at triggering cognitive conflict to encourage students to question the properties they observe visually, thus forcing them to shift to more abstract reasoning. Supporting deductive reasoning to provide informal and semi-formal proof practice before demanding a fully formal two-column proof, to bridge the gap between Level 2 and Level 3 (Çalışkan-Dedeoğlu, 2022)

4. CONCLUSION

The findings of this study indicate that students' level of understanding of Euclidean geometry is influenced by how they cognitively process information. Strong mental representations, logical thinking skills, and visualization skills are important factors in achieving higher levels of understanding. These results align with the theory that geometry learning must be accompanied by a balanced visual, verbal, and symbolic approach. This study also emphasizes the importance of evaluation based on the thinking process, not just the final mathematical result.

1. **Dominant Cognitive Level:** Students of the Mathematics Education Study Program show a strong tendency to be at the low level of Van Hiele's Geometry thinking, namely **Level 0 (Visualization)** and **Level 1 (Analysis)**, which is characterized by adequate ability in mastering basic concepts but failure in tasks that require high reasoning
2. **Evidence Gap:** The main obstacle for students lies in their inability to make the cognitive transition towards **Level 3 (Formal Deduction)**, proven by 72.7% of students experiencing difficulties in compiling logical proofs.

3. **Instructional Suggestions:** It is necessary to plan a learning strategy for Euclidean Geometry that is structured **and phased**, using Van Hiele's framework, to intentionally encourage students' cognitive development from the Analysis stage to the Formal Deduction stage. Further research is recommended to test the effectiveness of specific didactic interventions to facilitate this transition to cognitive levels.

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