Application of the Pythagorean Theorem as a Quick Solution in Solving Relativistic Momentum Problems

Umi Uswatun Khasanah[1], Halimatus Sa'diyah[2], Firdaus Nur Rahmat[3], Novi Fitri Rahmadani[4], Bambang Supriadi[5].
[1,2,3,4,5] Program Studi Pendidikan Fisika, Fakultas Keguruam dan Ilmu Pendidikan, Universitas Jember

Abstract
The Pythagorean theorem is a fundamental concept in mathematics that is often used to solve various geometric problems. In physics studies, especially in relativistic momentum analysis, the application of this theorem can be a fast and effective solution. This article explores the application of the Pythagorean theorem in the context of relativistic momentum, where the speed and energy of objects approach the speed of light. By exploiting the quadratic relationship between the components of the relativistic momentum and energy vectors, the Pythagorean theorem allows for simpler and more efficient calculations. This research identifies the relationship between the Pythagorean theorem equation and the relativistic momentum equation through several steps. The first step involves calculating the Pythagorean theorem equation. The second step is to calculate Einstein's special relativity equations. The final step is to analyze the relationship between the two equations. In this case, the relativistic and Pythagorean momentum equations have similarities and one of the formulas can be used.

Kata Kunci:*Teorema PhytagorasMomentum RelativitasRelativitas Khusus*

INTRODUCTION

The theory of relativity comes from the word relative, which means showing something that is relative or the opposite of absolute. The Theory of Relativity was developed by scientist Albert Einstein in 1905 for special relativity and in 1916 for general relativity. The emergence of the theory of Relativity was due to the consequences of the absence of a universal reference frame. Albert Einsein developed the Special Theory of Relativity which analyzes universal reference frames which are reference frames that move at a constant speed with respect to other frames [1].

Based on research conducted by [2], the application of the Pythagorean method to solve special relativity problems is as a fast and simple solution to solve Einstein's special relativity problem which is obtained from the relationship between the equations in Einstein's special theory of relativity and the equations in the Pythagorean theorem. The Pythagorean method solution for solving Einstein's special relativity problem is Einstein's special relativity problem which previously had many equations so that the solution took a long time and was quite difficult to do, so by using the Pythagorean method Einstein's special relativity problem can be done quickly and easily [3].

The results of other research [4] show that by using the Pythagorean method and several mathematical equations, Einstein's special relativity problems relating to relativistic mass, length contraction and time expansion can be solved easily and quickly. The application of the Pythagorean method in solving Einstein's special relativity problems also has a significant effect on learning outcomes [5]. In Korkmaz's research [6], the Pythagorean method can be used for length contraction, time expansion, and mass concepts, but has not been used to solve relativistic momentum. Therefore, this research discusses the application of the Pythagorean theorem as a quick solution in solving relativistic momentum problems.

According to [7]. Classical physics defines momentum as a particle that has a mass of m and is moving with a speed of v. However, momentum in classical physics has limitations, namely that it cannot be used when objects move at speeds close to the speed of light or what is usually called relativistic momentum. This can happen because in classical mechanics when an object moves at a speed much less than the speed of light then the amount of momentum before and after the collision will have the same value, whereas in relativistic momentum this does not apply because in relativistic momentum the relativistic mass is used so that the amount of momentum before and after the collision the value will not be the same [8].

The Pythagorean theorem is one of the basic concepts in mathematics, especially in the field of geometry. This theorem is named after the ancient Greek philosopher and mathematician, Pythagoras. The Pythagorean Theorem states that in a right triangle, the square of the length of the hypotenuse (the hypotenuse opposite the right angle) is equal to the sum of the squares of the lengths of the two legs of the sides that form the right angle [9].

The Pythagorean Theorem has many practical applications in various fields such as engineering, architecture, physics, and even art. For example, in building engineering, this theorem is used to ensure that building angles are truly right-angled. In addition, this theorem also forms the basis for further concepts in mathematics, including trigonometry and vector analysis [. Pythagoras and his students in the group known as the Pythagoreans also believed that numbers had mystical meaning, and this theorem became one of the pillars in their understanding of the relationship between numbers and geometric shapes [10].

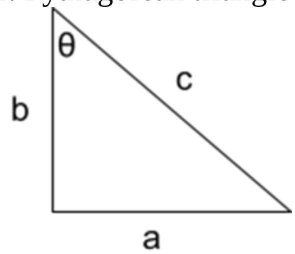
Relativistic momentum is a concept introduced in the special theory of relativity by Albert Einstein. In classical physics, momentum is calculated as the product of an object's mass and velocity. However, as the speed of the object approaches the speed of light, this definition is no longer accurate and must be modified to include relativistic effects. The relativistic factor known as the Lorentz factor takes into account the increase in the relative mass of an object as its speed approaches the speed of light [11]. This means that relativistic momentum depends not only on mass and speed, but also on the speed of the object relative to the speed of light. In the four-dimensional energy-momentum space, the relationship between the total energy, momentum and rest mass of an object can be likened to the relationship between the sides of a triangle in the Pythagorean Theorem [12]. The total energy of a particle in relativity is the combination of its kinetic energy and its rest energy, which together form a more complex relationship than that found in classical physics[13].

RESEARCH METHOD

This research identifies the relationship between the Pythagorean theorem equation and the relativistic momentum equation through several steps. The first step involves calculating the Pythagorean theorem equation. The second step is to calculate Einstein's special relativity equations. The final step is to analyze the relationship between the two equations to find out how they are related to each other. The calculation steps for the Pythagorean theorem equation and relativistic momentum are as follows:

1. Pythagorean Theorem Equation:

Figure 1. Pythagorean triangle



In the Pythagorean identity the angles in the triangle can be defined as:

$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{b}{c}$$

The Pythagorean Theorem states that the square of the length of the hypotenuse is equal to the sum of the squares of the height of a right triangle and its base. The Pythagorean theorem can also be obtained from the trigonometric relationship between sine θ and cosine θ with the following equation:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

2. Relativistic Momentum Equation:
The relativistic mass equation is

$$m' = \frac{m}{\sqrt{1 - v^2/c^2}}$$

So the relativistic momentum equation becomes

$$p = m'v$$

$$p = \frac{m}{\sqrt{1 - v^2/c^2}}(v)$$

$$p = \frac{mv}{\sqrt{1-v^2/c^2}} \text{or} p = \frac{\text{p0}}{\sqrt{1-v^2/c^2}}$$

Where v is the speed of the particle, m is the mass of the object when it is at rest [14].

3. The relationship between the Pythagorean theorem and relativistic momentum:

$$p = \frac{p0}{\sqrt{1 - v^2/c^2}}$$

$$p^2 = \frac{p0^2}{1 - v^2/c^2}$$

$$1 - v^2/c^2 = \frac{p0^2}{p^2}$$

$$1 = \frac{p0^2}{p^2} + \frac{v^2}{c^2}$$

It turns out that the above equation is identical to the Pythagorean identity

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

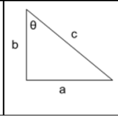
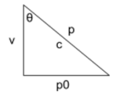
So the relativistic momentum equation using the Pythagorean identity can be written as

$$\left(\frac{p0}{p}\right)^2 + \left(\frac{v}{c}\right)^2 = 1$$

RESEARCH RESULTS AND DISCUSSION

The following are the results of the analysis of the Pythagorean Theorem equation with the relativistic momentum equation. When analyzing the time dilation equation, analysis can be done by connecting the time dilation equation with the Pythagorean equation. This equation is obtained by determining the variables of the time dilation equation with the Pythagorean theorem expression variables, namely P0= a, v=b, delta P and c = c. In the same length contraction equation, the same thing is determined, namely assuming the length contraction equation variable with the Pythagorean theorem equation variable.After connecting the Pythagorean identity equation with the relativistic momentum equation, it turns out that both have similarities, such as:

Table 1. Similarities between the two

No.	Persamaan Momentum Relativistik	Identitas Pitagoras	Gambar Identitas Pitagoras	Gambar Momentum Relativistik
1.	$\left(\frac{p0}{p}\right)^2 + \left(\frac{v}{c}\right)^2 = 1$	$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$		

.

Based on the results of the analysis of the Pythagorean theorem equation with Einstein's special relativity equation. When analyzing the time dilation equation, analysis can be carried out by connecting the time dilation equation with relativistic momentum

$$\left(\frac{p0}{p}\right)^2 + \left(\frac{v}{c}\right)^2 = 1$$

and the Pythagorean Identity, it can be seen that relativistic and Pythagorean momentum have similarities in these equations. If it is depicted it will be seen in the table. With this, the relativistic and Pythagorean momentum equations have similarities and one of the formulas can be used. The results in this research prove that the relativistic momentum equation and the Pythagorean identity are the same and have been proven. $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$

The following is an example of a relativistic momentum problem and its solution using the Pythagorean theorem:

An electron with a rest mass of 9.1 x 10-31 kg moves at a speed of 0.8 c. Calculate the value of the relativistic momentum! [15].

Solution:

Calculate using the usual equation:

$$p = \frac{p0}{\sqrt{1 - v^2/c^2}} = p = \frac{m0v}{\sqrt{1 - v^2/c^2}}$$

$$p = \frac{9{,}1 \times 10^{-31}(0{,}8c)}{\sqrt{1 - 0{,}8c^2/c^2}}$$

$$p = \frac{7{,}28c \times 10^{-31}}{\sqrt{1 - 0{,}64}}$$

$$p = \frac{7{,}28c \times 10^{-31}}{0{,}6}$$
$$p = 12{,}13 \times 10^{-31}$$
$$p = 12{,}13 \times 10^{-31}\ (3 \times 10^{8})$$
$$p = 36{,}39 \times 10^{-23} kgm/s$$

Calculate usingPythagorean equation:

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$
$$\left(\frac{7{,}28 \times 10^{-31}}{p}\right)^2 + \left(\frac{0{,}8c}{c}\right)^2 = 1$$
$$\left(\frac{53c \times 10^{-62}}{p^2}\right) + (0{,}64) = 1$$
$$\left(\frac{53c \times 10^{-62}}{p^2}\right) = 0{,}36$$
$$\left(\frac{53c \times 10^{-62}}{0{,}36}\right) = p^2$$
$$147{,}21c \times 10^{-62} = p^2$$
$$12{,}13c \times 10^{-31} = p$$
$$36{,}39 \times 10^{-23} = p$$

CONCLUSION

Based on the results of the analysis of Einstein's special relativity equation, a new equation has been obtained. After analysis, it shows that the relativistic momentum equation and the Pythagorean theorem have the same form. The similarity in the form of the equation can make the Pythagorean theorem a simple solution to solving relativistic momentum problems.

ACKNOWLEDGEMENT

Thank you to the article writing team friends who played an active role in preparing this article. We would also like to thank the course lecturers who have guided us so that this article was completed well.

BIBLIOGRAPHY

[1] Kurnia, A. (2021). Konsep Pemahaman Teori Relativitas Khusus Einstein Tentang Pemuaian Waktu. Konsep Pemahaman Teori Relativitas, 15(2), 173–179.

[2] Sanusi, E. N., & Haryadi, R. (2022). Studi Literatur Metode Pythagoras Dalam Penyelesaian Soal-Soal Teori Relativitas Khusus Einstein. *EduFisika: Jurnal Pendidikan Fisika*, *7*(2), 242–244. https://doi.org/10.59052/edufisika.v7i2.22309

[3] Supriadi, B., Istighfarini, E. T., Amelia, F., Mardhiana, H., & Sari, P. (2022). the Right Triangle Theorem in Comparison of Relavistic Energy. *Jurnal Riset Fisika Edukasi Dan Sains*, *9*(1), 34–41. https://doi.org/10.22202/jrfes.2022.v9i1.5751

[4] Khasanah, R. U., Supriadi, B., & Prastowo, S. H. B. (2019). Aplikasi Metode Pythagoras Dalam Penyelesaian Soal- Soal Relativitas Khusus Einstein Terhadap Hasil. *Jurnal Pembelajaran Fisika*, *8*(1), 24–31.

[5] Supriadi, B., Bektiarso, S., Damasari, A. F., Ramadhani, P. I., Febrianti, T. R., & Lubna, L. (2022). Respon Siswa Terhadap Metode Pythagoras Sebagai Alternatif Penyelesaian Soal Energi Relativistik. *ORBITA: Jurnal Pendidikan Dan Ilmu Fisika*, *8*(1), 128. https://doi.org/10.31764/orbita.v8i1.8582

[6] Korkmaz, S. D., Aybek, E. C., & Örücü, M. (2016). Special relativity theorem and Pythagoras's magic. *Physics Education*, *51*(2). https://doi.org/10.1088/0031-9120/51/2/025010

[7] Alivia, H., Wahyudianti, R., Hidayati, H., Imaniyah, K., & Prastowo, S. H. B. (2023). Analisis Penguasaan Konsep Fisika Materi dan Perubahannya Melalui Penyelesaian Soal Isomorfik pada Siswa Kelas VII MTS Zainul Bahar. *DIAJAR: Jurnal Pendidikan Dan Pembelajaran*, *2*(3), 354–358. https://doi.org/10.54259/diajar.v2i3.1695

[8] Supriadi, B., Harijanto, A., Widyawati, T., Alisia, N., Arifin, M. M., & Fikri, A. M. K. (2019). Pythagoras method to complete einstein special relativity

issues. *Journal of Physics: Conference Series*, *1211*(1). https://doi.org/10.1088/1742-6596/1211/1/012050

[9] Supriadi, B., Royani, S. N. M., Nanda, A. A. T., Elisa, E., Jannah, E. M., & dan Maryani. (2023). *Pythagorean Theorem for Solving Simple RLC Circuit Problems*. Atlantis Press International BV. https://doi.org/10.2991/978-94-6463-138-8_12

[10] Rahmat Penius Halawa. (2024). Analisis Pemahaman Konsep Matematika Siswa Pada Materi Teorema Phytagoras Ditinjau Dari Motivasi Belajar Kelas Viii Smp Negeri 1 Lolofitu Moi. *Afore : Jurnal Pendidikan Matematika*, *3*(1), 26–40. https://doi.org/10.57094/afore.v3i1.1706

[11] Hermanto, A., Hanang, R., Budhi, S., Fisika, D., Matematika, F., & Mada, U. G. (2024). *Jurnal Fisika A Critical Examination on the Lorentz Contraction of a Rod*. *14*(1), 9–14.

[12] Okun, L. B. (2009). The theory of relativity and the Pythagorean theorem. *Energy and Mass in Relativity Theory*, *622*, 285–304. https://doi.org/10.1142/9789812814128_0030

[13] Dima, A. S. G., & Lalus, H. F. (2024). Review Nilai Rata-Rata Operator Gaya Potensial Step Pada Partikel Satu Dimensi Dalam Tinjauan Persamaan Schrödinger, Klein-Fock-Gordon, dan Dirac. *Journal Online of Physics*, *9*(2), 112–121. https://doi.org/10.22437/jop.v9i2.32168

[14] Djudin, T. (2021). *Pengantar Fisika Modern.* Sleman: Penerbit Deepublish.

[15] Surya, Y. (2009). FISIKA MODERN-Persiapan Olimpiade Fisika. Tangerang: KANDEL.